

Module-7:

Input Modeling- Data collection, Identifying the Distribution with Data: Histograms, Selection of the Appropriate Family of Distributions, Quantile-Quantile Plots.100 Parameter Estimation: Sample Mean and Sample Variance and various biased and unbiased Estimators. Goodness of Fit Tests applied to Simulation inputs: Chi-Square and Chi-Square with Equal Probabilities, Kolmogorov-Smirnov Tests, p- Values and Best Fits. Verification and Validation of Simulation Models- Verification and Validation of Simulation Models. Calibration and Validation: Face Validity, Validation of Assumptions, Input-Out Transformation Validation.

Input Modeling

Collect and analyze input data so that the simulation can be fed with appropriate data. This is a very important task. Without proper input data, the good simulation model won't generate correct, appropriate result.

There are following steps in the development of a useful model of input data:

1. Collect data from the real system of interest. This often requires a substantial time and resource commitment.
2. Unfortunately, in some situations it is not possible to collect data (for example, when time is extremely limited, when the input process does not yet exist, or when laws or rules prohibit the collection of data).
3. When data are not available, expert opinion and knowledge of the process must be used to make educated guesses.
4. Identify a probability distribution to represent the input process.
5. When data are available, this step typically begins with the development of a frequency distribution, or histogram, of the data.
6. Given the frequency distribution and a structural knowledge of the process, a family of distributions is chosen.
7. Choose parameters that determine a specific instance of the distribution family. When data are available, these parameters may be estimated from the data.

8. Evaluate the chosen distribution and the associated parameters for goodness of fit.
9. Goodness of fit may be evaluated informally, via graphical methods, or formally, via statistical tests.
10. The chi-square and the Kolmogorov-Smirnov tests are standard goodness-of-fit tests. If not satisfied that the chosen distribution is a good approximation of the data, then the analyst returns to the second step, chooses a different family of distributions, and repeats the procedure.
11. If several iterations of this procedure fail to yield a fit between an assumed distributional form and the collected data, the empirical form of the distribution may be used.

Identifying the Distribution of Your Data

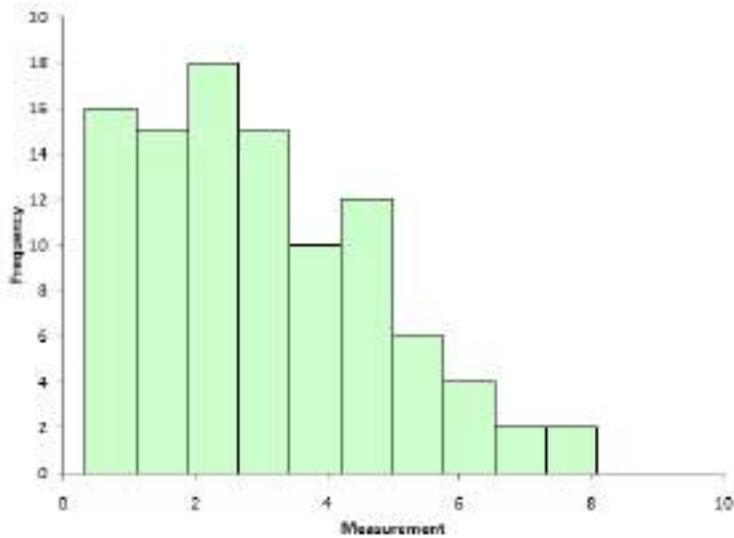
Distribution fitting is the process used to select a statistical distribution that best fits a set of data. Examples of statistical distributions include the normal, Gamma, Weibull and Smallest Extreme Value distributions. In the example above, you are trying to determine the process capability of your non-normal process.

Histograms

- A frequency distribution or histogram is useful in identifying the shape of a distribution.
- A histogram is constructed as follows.
 1. Divide the range of the data into intervals (intervals are usually of equal width).
 2. Label the horizontal axis to conform to the intervals selected.
 3. Determine the frequency of occurrences within each interval.
 4. Label the vertical axis so that the total occurrences can be plotted for each interval.
 5. Plot the frequencies on the vertical axis.

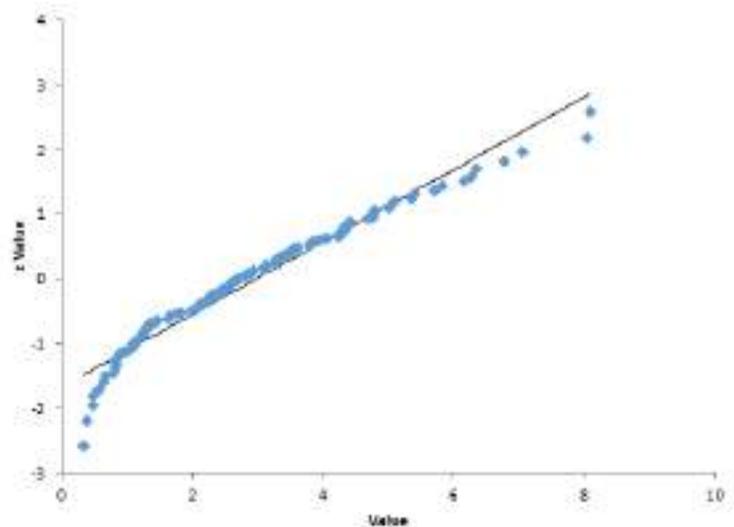
Suppose we have sample of 100 data points. You can download the data used **at this link**. A histogram (Figure 1) shows that the data are not normally distributed.

- **Figure 1: Histogram of Our Data**



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- The normal probability plot is shown in Figure 2. The data do not lie close to the straight line. The p-value for the Anderson-Darling statistic is 0.01, which is small. This confirms that the data are not normally distributed. For more information on the normal probability plot and the Anderson-Darling statistic, please see [this publication](#).

- **Figure 2: Normal Probability Plot of Our Data**



DISTRIBUTION FITTING FOR OUR DATA

- The next step is to fit the data to various distributions. Most software packages have numerous distributions that can be tested against the data. SPC for Excel was used to fit the various distributions. The output will be shown in three parts. The first part shows the parameters that were estimated for each distribution using the MLE method. These parameters are given Table 1.

- Table 1: Parameter Estimates from the Distribution Fitting**

Distribution	Location	Shape	Scale	Threshold
Weibull		1.729	3.342	
Weibull - Three Parameter		1.506	3.006	0.253
Gamma		2.446	1.216	
Gamma - Three Parameter		2.142	1.33	0.126
Largest Extreme Value	2.145		1.424	
LogNormal - Three Parameter	1.387		0.416	-1.379
LogNormal	0.872		0.719	
LogLogistic - Three Parameter	1.309		0.27	-1.058
LogLogistic	0.933		0.411	
Exponential - Two Parameter			2.646	0.329
Normal	2.975		1.78	
Logistic	2.848		1.019	
Exponential			2.975	

Distribution	Location	Shape	Scale	Threshold
Smallest Extreme Value	3.917		1.988	

- Not all distributions have the same parameters. For example, the normal distribution is described by the location and the scale while the Gamma distribution is described by the shape and scale. The parameters in Table 1 minimized the negative log-likelihood for each distribution. For the Weibull distribution, the shape parameter was estimated to be 1.729 and the scale parameter estimated to be 3.342. These two parameters minimized the negative log-likelihood for the Weibull distribution.
- The data in Table 1 are actually sorted by which distribution fits the data best. The next section describes how this was determined.

Selecting the Family of Distribution

There are many different distributions that may fit into a specific simulation task. Though exponential, normal and Poisson distributions are the ones used most often, others such as gamma and Weibull distributions are useful and important as well.

Here is a list of commonly used distributions.

Binomial

Models the number of successes in n trials, when the trials are independent with common success probability, p .

Negative Binomial including the geometric distribution

Models the number of trials required to achieve k successes.

Poisson

Models the number of independent events that occur in a fixed amount of time or space.

Normal

Models the distribution of a process that can be thought of as the sum of a number of component processes.

Log-normal

Models the distribution of a process that can be thought of as the product of a number of component processes.

Exponential

Models the time between independent events, or a process time which is memoryless.

Gamma

An extremely flexible distribution used to model non-negative random variables.

Beta

An extremely flexible distribution used to model bounded random variables.

Erlang

Models processes that can be viewed as the sum of several exponentially distributed processes.

Weibull

Models the time-to-failure for components.

Discrete or Continuous Uniform

Models complete uncertainty, since all outcomes are equally likely.

Triangular

Models a process when only the minimum, most-likely, and maximum values of the distribution are known.

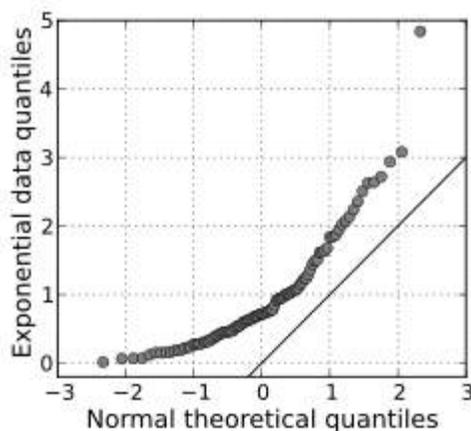
Empirical

Resamples from the actual data collected.

Quantile-Quantile Plots

A quantile-quantile (or Q-Q) plot visually compares two probability distributions by plotting a set of matching quantile values for both.

Q Q Plots (Quantile-Quantile plots) are plots of two quantiles against each other. A quantile is a fraction where certain values fall below that quantile. For example, the median is a quantile where 50% of the data fall below that point and 50% lie above it. The purpose of Q Q plots is to find out if two sets of data come from the same distribution. A 45 degree angle is plotted on the Q Q plot; if the two data sets come from a common distribution, the points will fall on that reference line.



The image above shows quantiles from a theoretical normal distribution on the horizontal axis. It's being compared to a set of data on the y-axis. This particular type of Q Q plot is called a **normal quantile-quantile (QQ) plot**. The points are not clustered on the 45 degree line, and in fact follow a curve, suggesting that the sample data is not normally distributed.

How to Make a Q Q Plot

Sample question: Do the following values come from a normal distribution?

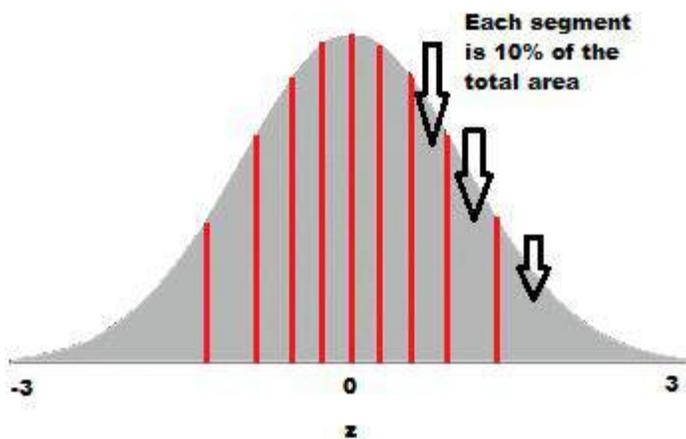
7.19, 6.31, 5.89, 4.5, 3.77, 4.25, 5.19, 5.79, 6.79.

Step 1: **Order the items from smallest to largest.**

- 3.77

- 4.25
- 4.50
- 5.19
- 5.89
- 5.79
- 6.31
- 6.79
- 7.19

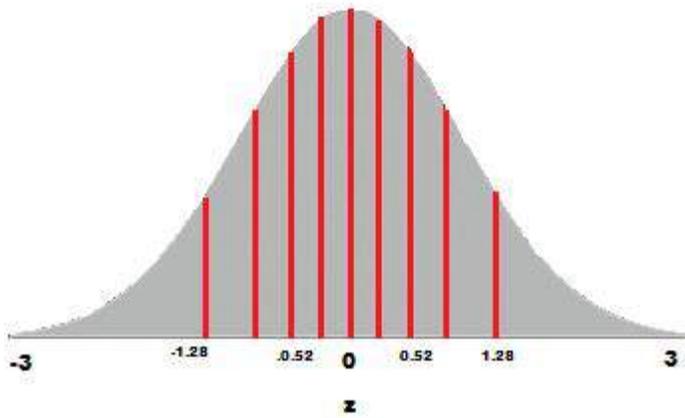
Step 2: Draw a normal distribution curve. Divide the curve into $n+1$ segments. We have 9 values, so divide the curve into 10 equally-sized areas. For this example, each segment is 10% of the area (because $100\% / 10 = 10\%$).



Step 3: Find the z-value (cut-off point) for each segment in Step 3. These segments are *areas*, so refer to a z-table (or use software) to get a z-value for each segment.

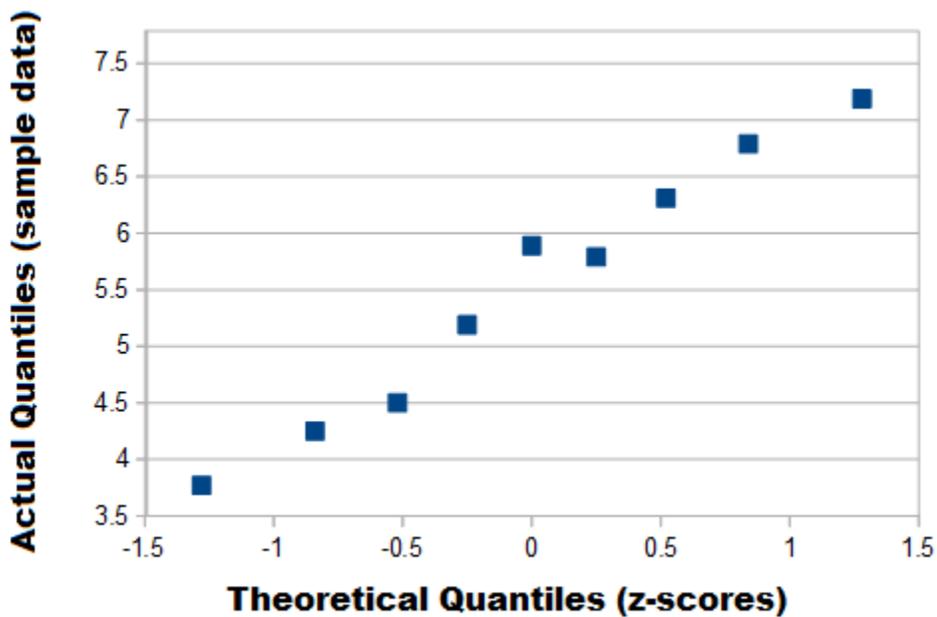
The z-values are:

- 10% = -1.28
- 20% = -0.84
- 30% = -0.52
- 40% = -0.25
- 50% = 0
- 60% = 0.25
- 70% = 0.52
- 80% = 0.84
- 90% = 1.28
- 100% = 3.0



A few of the z-values plotted on the graph.

Step 4: Plot your data set values (Step 1) against your normal distribution cut-off points (Step 3). I used Open Office for this chart:



The (almost) straight line on this q q plot indicates the data is approximately normal.

Parameter Estimation

Parameter Estimation is a branch of statistics that involves using sample data to estimate the parameters of a distribution.

Methods of Parameter Estimation

The techniques used for parameter estimation are called estimators.

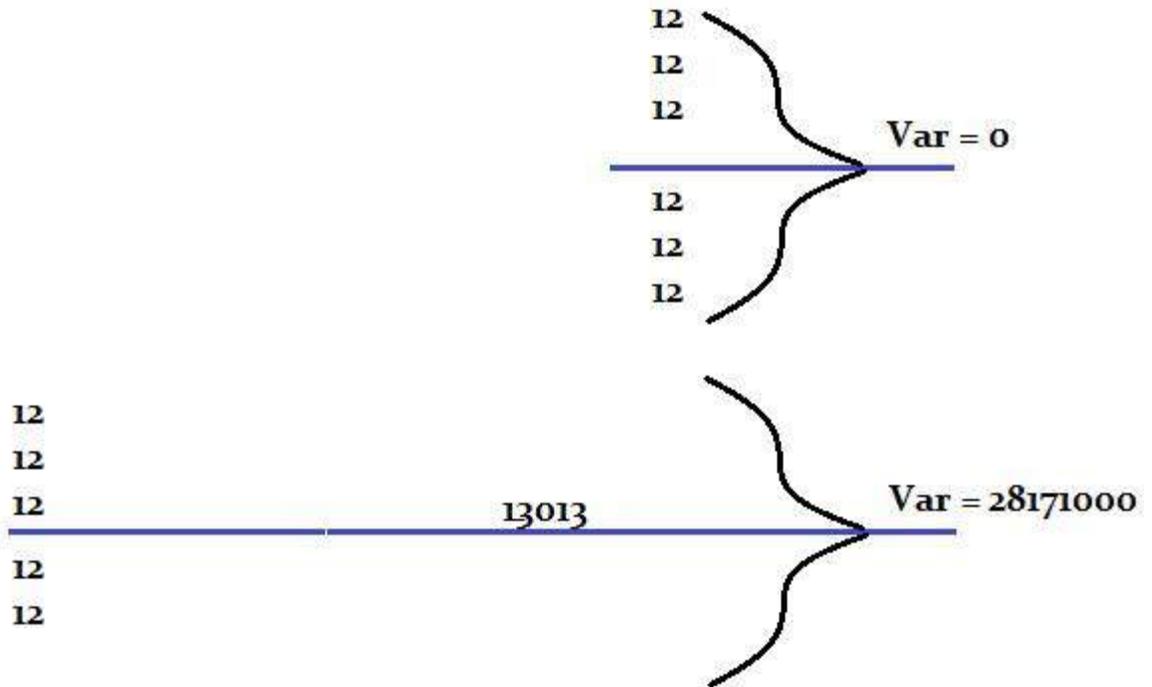
Some estimators are:

- **Probability Plotting:** A method of finding parameter values where the data is plotted on special plotting paper and parameters are derived from the visual plot
- **Rank Regression (Least Squares):** A method of finding parameter values that minimizes the sum of the squares of the residuals.
- **Maximum Likelihood Estimation:** A method of finding parameter values that, given a set of observations, will maximize the likelihood function.
- **Bayesian Estimation Methods:** A family of estimation methods that tries to minimize the posterior expectation of what is called the utility function. In practice, what this means is that existing knowledge about a situation is formulated, data is gathered, and then posterior knowledge is used to update our beliefs.

Qualities of Estimators

If the value an estimator estimates for the parameter, θ' , always converges to the actual parameter value θ as the quantity of data used for parameter estimation increases, we say an estimator is **consistent**.

The bias of an estimator is the deviation of the expectation from the actual true value. If, for a given estimator, the bias is zero, we say that that estimator is **unbiased**.



Variance tells you how spread out a data set is.

A third statistic that tells us about the reliability of an estimator is the variance. If an estimator has lower variance than another we say it is more **efficient**, and we can calculate the **efficiency** of estimator p relative to estimator q as $(\text{Var}(\theta'_q))/\text{Var}(\theta'_p)$.

- After a family of distribution has been selected such as Poisson, Normal, Geometric ..., the next step is to estimate the parameters of the distribution.
- Sample mean and sample variance can be used to estimate the parameters in a distribution.
 - Let X_1, X_2, \dots, X_n be the sample of size n .
 - The sample mean is

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

- The sample variance is

$$S^2 = \frac{\sum_{i=1}^n X_i^2 - n\bar{X}^2}{n-1}$$

- If the data are discrete and grouped in a frequency distribution, then we can re-write the equations as

$$\bar{X} = \frac{\sum_{j=1}^k f_j X_j}{n}$$

and

$$S^2 = \frac{\sum_{j=1}^k f_j X_j^2 - n\bar{X}^2}{n-1}$$

- Example 10.5 on page 368
- If the data are continuous, we ``discretize" them and estimate the mean

$$\bar{X} \approx \frac{\sum_{j=1}^c f_j m_j}{n}$$

and the variance

$$S^2 \approx \frac{\sum_{j=1}^c f_j m_j^2 - n\bar{X}^2}{n-1}$$

where f_j is the observed frequency in the j th class interval, m_j is the midpoint of the j th interval, and c is the number of class intervals.

- Example 10.6 on page 369
- A few well-established, suggested estimators are listed in Table 10.3 on page 370, followed by examples. They come from theory of statistics.
- The examples include Poisson Distribution, Uniform Distribution, Normal Distribution, Exponential Distribution, and Weibull Distribution.

Goodness-of-Fit Tests

Discrete probability distributions are based on discrete variables, which have a finite or countable number of values. In this post, I show you how to perform goodness-of-fit tests to determine how well your data fit various discrete probability distributions.

The sum of all probabilities must equal 1. In contrast, continuous distributions are based on continuous variables and have an infinite number of possible values.

The following are examples of different types of discrete distributions.

- **Binary:** For each customer that enters a dealership, there are two possible outcomes—sale or no sale. Each outcome has a probability.
- **Poisson:** The number of cars that a dealership sells in a day can follow the Poisson distribution. You can create a table with the counts (0, 1, 2, 3, etc.) along with the probability for each daily count.
- **Categorical:** The color of the car is a categorical variable. You can list all possible colors along with their probabilities.
- Goodness-of-fit tests provide helpful guidance for evaluating the suitability of a potential input model.
- The tests depends heavily on the amount of data. If very little data are available, the test is unlikely to reject *any* candidate distribution (because not enough evidence to reject); if a lot of data are available, the test will likely reject *all* candidate distributions (because none fits perfectly).
- Failing to reject a candidate should be viewed as a piece of evidence in favor of that choice; while rejecting an input model is only one piece of evidence against the choice.

- Chi-square test is for large sample sizes, for both discrete and continuous distributional assumptions, when parameters are estimated by maximum likelihood.
 - Arranging the n observations into a set of k class intervals or cells.
 - The test statistic

$$x_0^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

where O_i is the observed frequency in the i th class interval and E_i is the expected frequency in that class interval.

- The x_0^2 approximately follows the chi-square distribution with $k-s-1$ degrees of freedom, where s represents the number of parameters of the estimated distribution. E.g Poisson distribution has $s = 1$, normal distribution has $s=2$.
- The hypothesis

H_0 :

the random variable, X , conforms to the distributional assumption with the parameter(s) given by the parameter estimate(s)

H_1 :

the random variable X does not conform the distribution

- The critical value $x_{\alpha, k-s-1}^2$ is found in Table A.6. H_0 is rejected if $x_0^2 > x_{\alpha, k-s-1}^2$.
- The choice of k , the number of class intervals, see Table 10.5 on page 377.
- Example 10.13 on page 377.
- Chi-square test with equal probabilities:
 - If a continuous distributional assumption is being tested, class intervals that are equal in probability rather than equal in width of interval should be used.
 - Example 10.14: Chi-square test for exponential distribution (page 379)

- test with intervals of equal probability (not necessary equal width)
- number of intervals less than or equal to $n/5$
- $n = 50$, so $k \leq 10$, according to recommendations in Table 10.5, 7 to 10 class intervals be used.
- Let $k = 8$, thus $p = 0.125$
- The end points for each interval are computed from the cdf for the exponential distribution

$$F(a_i) = 1 - e^{-\lambda a_i}$$

where a_i represents the end point of the i th interval.

- Since $F(a_i)$ is the cumulative area from zero to a_i ,
thus $F(a_i) = ip$

$$ip = 1 - e^{-\lambda a_i}$$

thus

$$a_i = -\frac{1}{\lambda} \ln(1 - ip) \quad i = 0, 1, \dots, k$$

regardless the value of λ , $a_0 = 0$ and $a_k = \infty$.

- With $\lambda = 0.084$ in this example and $k = 8$,

$$a_1 = -\frac{1}{0.084} \ln(1 - 0.125) = 1.590$$

continue with $i = 2, 3, \dots, 7$ results in 3.425, 5.595, 8.252, 11.677, 16.503, and 24.755.

- See page 379 and 380 for completion of the example.
- Example 10.15 (Chi-square test for Weibull distribution) on page 380
- Example 10.16 (Computing intervals for the normal distribution) on page 381
 - For the given data, using suggested estimator in Table 10.3 on page 370, we know (the original data was from Example 10.3 on page 360)

$$\mu = \bar{x} = 11.90$$

$$\sigma^2 = S^2 =$$

- Kolmogorov-Smirnov Goodness-of-fit test
 - Chi-square test heavily depends on the class intervals. For the same data, different grouping of the data may result in different conclusion, rejection or acceptance.
 - The K-S goodness-of-fit test is designed to overcome this difficulty. The idea of K-S test is from q-q plot.
 - The K-S test is particularly useful when sample size are small and when no parameters have been estimated from the data.
 - Example 10.7 on page 383, using the method described in Section 8.4.1 on page 299. A few notes:
 - If the interarrival time is exponentially distributed, the arrival times are uniformly distributed on $(0, T]$

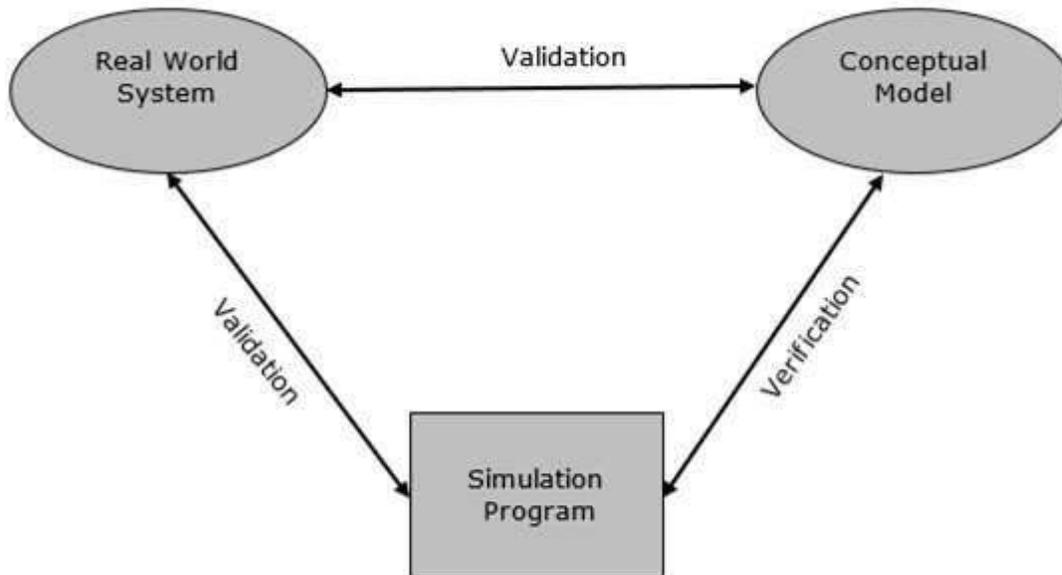
Verification and Validation of Simulation Models

Verification and validation of computer simulation models is conducted during the development of a simulation model with the ultimate goal of producing an accurate and credible model. Simulation models are increasingly being used to solve problems and to aid in decision-making. The developers and users of these models, the decision makers using information obtained from the results of these models, and the individuals affected by decisions based on such models are all rightly concerned with whether a model and its results are "correct".^[3] This concern is addressed through verification and validation of the simulation model.

Simulation models are approximate imitations of real-world systems and they never exactly imitate the real-world system. Due to that, a model should be verified and validated to the degree needed for the model's intended purpose or application

Validation and verification are the two steps in any simulation project to validate a model.

- **Validation** is the process of comparing two results. In this process, we need to compare the representation of a conceptual model to the real system. If the comparison is true, then it is valid, else invalid.
- **Verification** is the process of comparing two or more results to ensure its accuracy. In this process, we have to compare the model's implementation and its associated data with the developer's conceptual description and specifications.



Verification & Validation Techniques

There are various techniques used to perform Verification & Validation of Simulation Model. Following are some of the common techniques –

Techniques to Perform Verification of Simulation Model

Following are the ways to perform verification of simulation model –

- By using programming skills to write and debug the program in sub-programs.
- By using “Structured Walk-through” policy in which more than one person is to read the program.
- By tracing the intermediate results and comparing them with observed outcomes.
- By checking the simulation model output using various input combinations.
- By comparing final simulation result with analytic results.

Techniques to Perform Validation of Simulation Model

Step 1 – Design a model with high validity. This can be achieved using the following steps –

- The model must be discussed with the system experts while designing.
- The model must interact with the client throughout the process.
- The output must supervised by system experts.

Step 2 – Test the model at assumptions data. This can be achieved by applying the assumption data into the model and testing it quantitatively. Sensitive analysis can also be performed to observe the effect of change in the result when significant changes are made in the input data.

Step 3 – Determine the representative output of the Simulation model. This can be achieved using the following steps –

- Determine how close is the simulation output with the real system output.
- Comparison can be performed using the Turing Test. It presents the data in the system format, which can be explained by experts only.
- Statistical method can be used for compare the model output with the real system output.

Model Data Comparison with Real Data

After model development, we have to perform comparison of its output data with real system data. Following are the two approaches to perform this comparison.

Validating the Existing System

In this approach, we use real-world inputs of the model to compare its output with that of the real-world inputs of the real system. This process of validation is straightforward, however, it may present some difficulties when carried out, such as if the output is to be compared to average length, waiting time, idle time, etc. it can be compared using statistical tests and hypothesis testing. Some of the statistical tests are chi-square test, Kolmogorov-Smirnov test, Cramer-von Mises test, and the Moments test.

Validating the First Time Model

Consider we have to describe a proposed system which doesn't exist at the present nor has existed in the past. Therefore, there is no historical data available to compare its performance with. Hence, we have to use a hypothetical system based on assumptions. Following useful pointers will help in making it efficient.

- **Subsystem Validity** – A model itself may not have any existing system to compare it with, but it may consist of a known subsystem. Each of that validity can be tested separately.
- **Internal Validity** – A model with high degree of internal variance will be rejected as a stochastic system with high variance due to its internal processes will hide the changes in the output due to input changes.
- **Sensitivity Analysis** – It provides the information about the sensitive parameter in the system to which we need to pay higher attention.
- **Face Validity** – When the model performs on opposite logics, then it should be rejected even if it behaves like the real system.

So far, we have discussed how to run simulation, how to generate and test random number generators, how to build input data models. This chapter discusses how one might verify and validate a simulation model.

The goal of validation process

- to produce a model that represents true system behavior close enough for the model to be used as a substitute for the physical system
- to increase to an acceptable level of credibility of the model

The verification and validation process consists of the following components.

1. Verification is concerned with building the model right.
2. Validation is concerned with building the right model.

Calibration and Validation of Models

- For a given program, while common sense verification is possible, strict verification of a model is intractable, very much similar to the proof of correctness of a program.
- Validation is a process of comparing the model and its behavior to the real system and its behavior.
- Calibration is the iterative process of comparing the model with real system, revising the model if necessary, comparing again, until a model is accepted (validated).
- Three-step approach in the validation process.
 1. Build a model that has high face validity.
 2. Validate model assumptions
 3. Compare the model input-output transformations to corresponding input-output transformation for the real system.

The three-step approach is discussed next.

Face Validity

A **model** that has **face validity** appears to be a reasonable imitation of a real-world system to people who are knowledgeable of the real world system. **Face validity** is tested by having users and people knowledgeable with the system examine **model** output for reasonableness and in the process identify deficiencies.

- Build a "reasonable model" on its face to model users who are knowledgeable about the real system being simulated.
- Do some "sanity check"

Validation of Model Assumptions

The assumptions identification and validation process is a critical part of effectively managing risks as part of a risk assessment for an organization's initiative, objective, or strategy. Assumptions are identified, classified by category (e.g., people, process, technology), and validated by the group. Each assumption is then assessed based on its impact and confidence level, then an action plan is developed in order to validate the high priority assumptions (i.e., high impact and low confidence).

The process consists of 4 simple steps:

1. Identify and categorize the assumptions made about your initiative.
2. Vote for the assumptions you agree apply to the initiative.
3. Rate each assumption based on its impact and your confidence level.
4. Discuss results, view alignment, and finalize an action plan to validate the assumptions.

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WHO

This model can be used by project managers, risk managers, department directors, and managers.

WHY

In order to maintain control over your initiatives, you need to ensure that you're addressing the various assumptions that are being made and monitoring them as part of an ongoing process. Without identifying and tracking these assumptions, your initiative could be impacted at any time, leaving you and your organization scrambling to get back on track. Allowing your stakeholders to participate in this process puts everyone on the same page and ensures that you're taking all the different perspectives into consideration.

Stakeholders feel secure knowing that their insight and rationale are heard and valued, and that everyone benefits from identifying and investigating assumptions, even if the facts show that they're wrong (McKinsey & Company, 2010): *“Push yourself and those around you to make assumptions explicit. Tease the assumptions out of arguments if you need to. Challenge whether you agree with the assumption or not. Step back and see if it is in fact accurate. This approach will create better debates, better analysis and certainly better decision making.”* (Kaplan, 2012)

Validating high priority assumptions on an ongoing basis will allow for quick course corrections to keep your plans on the path to success (McKinsey & Company, 2010): *“By identifying your [assumptions] early and devising ways to test hypotheses that will prove or refute them, you are in a position to learn whether or not your Plan A will work before you waste too much of your time, and your and your investors' money.”* (Mullins & Komisar, 2009)

HOW

- 1) **NOODLE & TAG: Identify the assumptions that your initiative's success depends on, then categorize each assumption using the following tags (can be customized):**
- 2) **COMBINE to eliminate duplicates and move forward with only unique assumptions.**
- 3) **VOTE for assumptions that you agree apply to your initiative.**
- 4) **MULTI-CRITERIA RATE each assumption based on Impact and Confidence. In the comments section, provide rationale for why each assumption was rated the way it was and attach any supporting documents to each tile.**
- 5) **SHARE AND DISCUSS RESULTS: Display the graph of results to view alignment. You can apply a heat map to the graph to easily identify the high priority assumptions (i.e., low confidence and high impact).**
- 6) **ACTION PLAN: Develop a specific action plan to validate the assumptions.**

RESULTS

- Valuable insight into the assumptions being made and the impact they would have if found to be false
- Shared stakeholder understanding and alignment on assumptions
- Action plan to validate each assumption made about the initiative

BENEFITS & IMPACT

This exercise will enable:

Quality – Integrate stakeholders' ideas and contributions into your assumptions identification process. This allows you to proactively manage assumptions and avoid surprises down the road.

Efficiency – Engage stakeholders at times that are convenient for them to contribute – 24/7.

Engagement – Create a rich discussion with stakeholders while analyzing and critically evaluating how different assumptions can impact your initiative.

Agility – Develop a shared understanding about your initiative's key assumptions, as well as an action plan to keep high priority assumptions in check.

Validation of Model Assumptions

- Model assumptions fall into two categories: structural assumptions and data assumptions.

- Structural assumptions deal with such questions as how the system operates, what kind of model should be used, queueing, inventory, reliability, and others.
- Data assumptions: what kind of input data model is? What are the parameter values to the input data model?

Validating Input-Output Transformations

Verification of models:

- Verification is defined as the process of correctly building a system model. Verification is done to ensure that:
- The model is programmed correctly. The algorithms have been implemented correctly.
- The model does not contain errors, oversights, or bugs. Verification ensures that the specifications is complete and that mistakes have not been made in implementing the model.
- Verification does not ensure the model: Solves an important problem. Meets a specified set of requirements. Correctly reflects the working of a real world process.
- The purpose of verification is to make sure that the conceptual model is reflected accurately in operational model. It asks questions regarding the correct implementation of the model, correct representation of input parameters and logical structure, etc.
- Verification is determining whether the simulation computer program performs as intended. Verification checks for translation of conceptual simulation model onto a correctly working program.

Validation of Models:

- The standard method to validate model is to construct a model of the existing system. Then, change the model appropriately in order to analyze each alternative.
- The model of the existing system can be validated by comparing its results against actual data obtained from the system under investigation. Goal of validation is to ensure that the simulation model is good enough so that it can be used to make decisions about systems that we ideally would like to work with.
- Ease or difficulty of the validation process depends on the complexity of the system being modelled and on whether a version of a system currently exists.
- A simulation model of a complex system can only be an approximation to the actual system, regardless of how much effort is put into development. There is no such thing as absolutely valid model!
- A simulation model is always developed for a particular purpose .a logbook of the simulation model's assumptions should be updated on a regular basis and eventually should form integral part of final report.
- A simulation model should be validated relative to those measures of performance that will be actually used for decision making.

Validating Input- Output Transformations

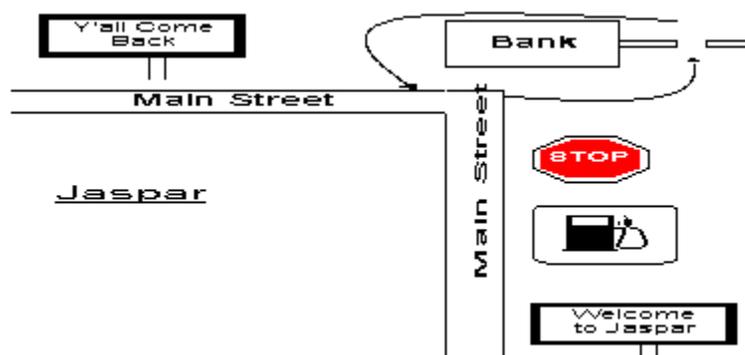
- The ultimate test of a model, and in fact the only objective test of the model as a whole is the model's ability to predict the future behavior of the real system when the model input data match the real inputs and

when a policy implemented in the model is implemented at some point in the system.

- The structure of the model should be accurate enough to make good predictions for the range of input sets of interest.
- We can see the outputs of the systems as being a functional transform of the inputs based on parameter settings. i.e. the model accepts values of input parameters and transforms them into suitable outputs measures of performances.

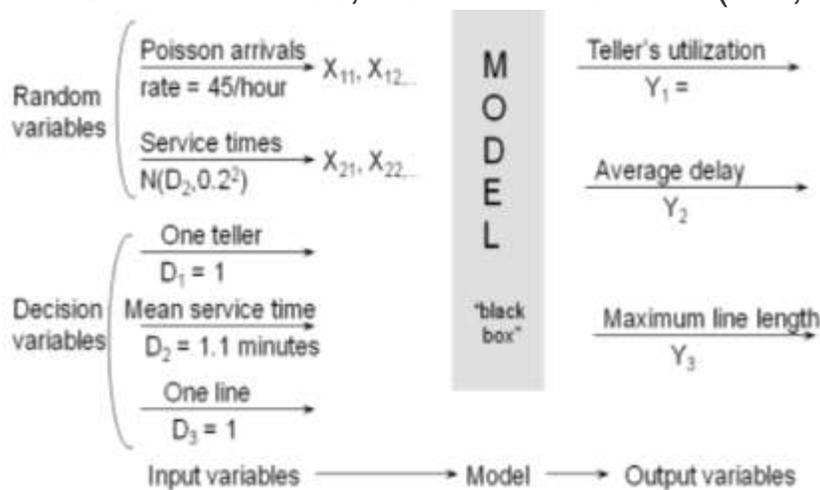
(Example) : The Fifth National Bank of Jaspar

- The Fifth National Bank of Jaspar, as shown in the next slide, is planning to expand its drive-in service at the corner of Main Street. Currently, there is one drive-in window serviced by one teller. Only one or two transactions are allowed at the drive-in window, so, it was assumed that each service time was a random sample from some underlying population. Service times $\{S_i, i = 1, 2, \dots, 90\}$ and inter arrival times $\{A_i, i = 1, 2, \dots, 90\}$



were collected for the 90 customers who arrived between 11:00 A.M. and 1:00 P.M. on a Friday. This time slot was selected for data collection after consultation with management and the teller because it was felt to be representative of a typical rush hour. Data analysis led to the conclusion that the arrival process could be modeled as a Poisson process with an arrival rate of 45 customers per hour; and that service times were approximately normally distributed with mean 1.1 minutes and standard deviation 0.2 minute. Thus, the model has two input variables:

1. Inter arrival times, exponentially distributed (i.e. a Poisson arrival process) at rate $\lambda = 45$ per hour.
2. Service times, assumed to be $N(1.1, (0.2)^2)$



Validation using Historical Input data

- An alternative to randomly generated data – don't mix different data sets
- File, Spreadsheet, or Database
 - $\{A_1, A_2, \dots, A_n\}$ & $\{S_1, S_2, \dots, S_n\}$
 - Feed data into the FEL

- Compare output to what happened in the real system
- May be able to use technology to collect historical data for use

Validation using a Turing Test

- What is the Turing Test?
- Generate 5 “fake” reports from simulation & mix with 5 real reports; ask experts if they can distinguish fake from real
- If cannot, then pass Turing Test!

Validating Input-Output Transformations

- View the model as a black box
- Feed the input at one end and examine the output at the other end
- Use the same input for a real system, compare the output with the model output
- If they fit closely, the black box seems working fine
- Otherwise, something is wrong